

## ДОМАШНЕЕ ЗАДАНИЕ № 3

### «Интегральное исчисление функции одной переменной»

I. Найти неопределенные интегралы. Правильность полученных результатов проверить дифференцированием.

$$1. 1) \int \frac{x dx}{7+x^2}; \quad 2) \int \frac{(x+18) dx}{x^2-4x-12};$$

$$3) \int (3-x) \cos x dx.$$

$$2. 1) \int \frac{dx}{\sin^2 \frac{x}{5}};$$

$$2) \int \frac{(x+4) dx}{x^2-2x-8};$$

$$3) \int x \ln(1-3x) dx.$$

$$3. 1) \int \frac{x dx}{\sqrt{5-x^2}};$$

$$2) \int \frac{(x+23) dx}{x^2+x-20};$$

$$3) \int x e^{-7x} dx.$$

$$4. 1) \int \frac{dx}{5x+3};$$

$$2) \int \frac{(x+12) dx}{x^2-x-6};$$

$$3) \int \arctg 4x dx.$$

$$5. 1) \int \sin(2-3x) dx;$$

$$2) \int \frac{(x+19) dx}{x^2-2x-15};$$

$$3) \int \sqrt{x^3} \ln x dx.$$

$$6. 1) \int e^{\frac{x}{4}-2} dx;$$

$$2) \int \frac{(5x+6) dx}{x^2+4x-12};$$

$$3) \int x \sin 5x dx.$$

$$7. 1) \int \frac{dx}{7+4x^2};$$

$$2) \int \frac{(5x-7) dx}{x^2-x-20};$$

$$3) \int (2x+5) \sin x dx.$$

$$8. 1) \int \frac{dx}{\cos^2 2x};$$

$$2) \int \frac{5x dx}{x^2+x-6};$$

$$3) \int \frac{\ln x}{\sqrt{x}} dx.$$

$$9. 1) \int \cos\left(\frac{x}{3}-4\right) dx;$$

$$2) \int \frac{(5x+2) dx}{x^2+2x-8};$$

$$3) \int \arcsin \frac{x}{3} dx.$$

$$10. 1) \int \frac{dx}{\sqrt[3]{(2x+1)^2}};$$

$$2) \int \frac{(5x+1) dx}{x^2+2x-15};$$

$$3) \int x e^{3x} dx.$$

$$11. 1) \int \frac{e^x dx}{\sqrt[3]{1-e^x}};$$

$$2) \int \frac{(19-4x) dx}{2x^2+x-3};$$

$$3) \int (5x-2) \ln x dx.$$

$$12. 1) \int x \sqrt{3-x^2} dx;$$

$$2) \int \frac{(2x+9) dx}{x^2+5x+6};$$

$$3) \int x \cos^2 2x dx.$$

$$13. 1) \int \frac{\arctg x dx}{1+x^2};$$

$$2) \int \frac{(x+9) dx}{x^2+2x-3};$$

$$3) \int \ln(3+x^2) dx.$$

$$14. 1) \int \sin 2x \sqrt{2-\cos^2 x} dx;$$

$$2) \int \frac{(2x+27) dx}{x^2-x-12};$$

$$3) \int x \arcsin x dx.$$

$$15. 1) \int \frac{\sin x dx}{1-\cos x};$$

$$2) \int \frac{(4x+31) dx}{2x^2+11x+12};$$

$$3) \int (2-x) \sin x dx.$$

$$16. 1) \int \frac{\sin x dx}{\cos^3 x};$$

$$2) \int \frac{(x+4) dx}{\sqrt{2-x-x^2}};$$

$$3) \int \arccos x dx.$$

17. 1)  $\int (3 - 4 \sin x)^{1/3} \cos x dx$ ; 2)  $\int \frac{(4x-3)dx}{x^2 - 2x + 6}$ ;

3)  $\int (1-x) \ln x dx$

18. 1)  $\int \frac{dx}{x \ln^2 x}$  2)  $\int \frac{(x-3) dx}{\sqrt{x^2 - 6x + 1}}$ ;

3)  $\int x^2 \sin x dx$ .

19. 1)  $\int (1-5x)^{19} dx$  2)  $\int \frac{5x+6}{\sqrt{2x^2 - 4x + 5}} dx$ .

3)  $\int e^{2x} \cos x dx$ .

20. 1)  $\int \frac{e^{-x} dx}{1+e^{-2x}}$  2)  $\int \frac{(x+7)dx}{x^2 + 8x + 1}$ ;

3)  $\int (x+1)e^x dx$

21. 1)  $\int x \cdot 5^{-x^2} dx$  2)  $\int \frac{(2x-1)dx}{x^2 - 3x + 3}$ ;

3)  $\int x \cdot 3^x dx$

22. 1)  $\int \frac{x^3 dx}{x^8 + 1}$  2)  $\int \frac{3x-2}{\sqrt{x^2 + x + 1}} dx$ .

3)  $\int x \arctg x dx$

23. 1)  $\int \frac{x dx}{3-2x^2}$  2)  $\int \frac{x+1}{\sqrt{5+x-x^2}} dx$ .

3)  $\int x^2 \cdot e^x dx$

24. 1)  $\int \frac{xdx}{(x^2+1)^2}$  2)  $\int \frac{x-8}{\sqrt{x^2+x+1}} dx$ .

3)  $\int x^3 \cdot e^{x^2} dx$

25. 1)  $\int \frac{xdx}{\sqrt{1-4x^2}}$  2)  $\int \frac{(2x+5)dx}{x^2 - 3x + 6}$ ;

3)  $\int \log_{1/2}(1+x) dx$

26. 1)  $\int \frac{e^x dx}{1+e^x}$

3)  $\int \ln(x^2 + 1) dx$

2)  $\int \frac{(7x-9)dx}{x^2 - 2x + 5}$ ;

27. 1)  $\int \operatorname{ctg}(2x+1) dx$

3)  $\int \arccos(2x-3) dx$ .

2)  $\int \frac{3x-2}{\sqrt{4x-3-x^2}} dx$ .

28. 1)  $\int e^x (\sin e^x) dx$

3)  $\int \frac{\ln x}{x^3} dx$ .

2)  $\int \frac{2x+5}{x^2 + 3x - 10} dx$

29. 1)  $\int \frac{(1+x)dx}{\sqrt{1-x^2}}$

3)  $\int x \cos(x+1) dx$ .

2)  $\int \frac{3-x}{4x^2 + 4x + 5} dx$

30. 1)  $\int e^{x^2} x dx$

3)  $\int x^3 \sin x^2 dx$ .

2)  $\int \frac{(x-5)dx}{2x^2 - 3x - 2}$ ;

31. 1)  $\int \frac{\sqrt{\arctg x} dx}{1+x^2}$

3)  $\int (x-2)a^{-x} dx$ .

2)  $\int \frac{(11x-1)dx}{x^2 - 5x + 4}$ ;

32. 1)  $\int \frac{\cos x dx}{\sqrt[3]{\sin^2 x}}$

3)  $\int (x-6) \sin 2x dx$ .

2)  $\int \frac{(4x+3)dx}{2x^2 - 6x - 5}$ ;

33. 1)  $\int x \cos 3x^2 dx$ .

3)  $\int x e^{-2x} dx$

2)  $\int \frac{x-3}{\sqrt{8+6x-9x^2}} dx$

34. 1)  $\int \frac{\sin x dx}{\cos^3 x}$

3)  $\int \log_5(3+2x) dx$

2)  $\int \frac{3x-2}{x^2 - 3x + 8} dx$

$$35. 1) \int \cos x \cdot e^{\sin x} dx. \quad 2) \int \frac{2x+1}{\sqrt{2-6x+x^2}} dx$$

$$3) \int x^2 \sin 2x dx.$$

$$36. 1) \int x e^{-x^2-3} dx \quad 2) \int \frac{3x+5}{2x^2+3x+1} dx$$

$$3) \int x \operatorname{arctg}(2x+1) dx$$

$$37. 1) \int \frac{x dx}{1+x^4} \quad 2) \int \frac{x+1}{x^2-10x+7} dx$$

$$3) \int x \lg(2x+5) dx$$

$$38. 1) \int \frac{\sin(\ln x) dx}{x} \quad 2) \int \frac{2x+3}{3x^2+3x+1} dx$$

$$3) \int \arcsin(1-x) dx$$

$$39. 1) \int x^2 \sqrt{x^3+11} dx \quad 2) \int \frac{(2x-7)}{\sqrt{x^2-x-20}} dx.$$

$$3) \int \arccos \frac{x}{\sqrt{2}} dx$$

$$40. 1) \int \frac{dx}{\cos^2 x \sqrt{1+\operatorname{tg} x}} \quad 2) \int \frac{(8x-17)}{\sqrt{x^2-x-12}} dx.$$

$$3) \int (8x-3) \ln 2x dx$$

$$41. 1) \int \frac{d(1+\ln x)}{\cos^2(1+\ln x)} \quad 2) \int \frac{(x-11)}{x^2+2x-3} dx$$

$$3) \int (x-3) \cos \frac{1}{2} x dx$$

$$42. 1) \int \operatorname{tg} \frac{x}{\sqrt{5}} dx \quad 2) \int \frac{(24x+1)}{x^2+4x-12} dx$$

$$3) \int x \sin^2 x dx$$

$$43. 1) \int \frac{\cos x dx}{\sqrt[3]{\sin^2 x}} \quad 2) \int \frac{(x-19)}{x^2-x+20} dx$$

$$3) \int (x+3)e^{-4x} dx$$

$$44. 1) \int \operatorname{ctg}(2x-5) dx \quad 2) \int \frac{(5x-8)}{\sqrt{x^2-x-12}} dx.$$

$$3) \int \operatorname{arctg} \frac{x}{\sqrt{11}} dx$$

$$45. 1) \int \frac{x^4 dx}{\sqrt{14+x^5}} \quad 2) \int \frac{(2x-19)}{x^2-2x+21} dx$$

$$3) \int (x+5) \ln(2x-1) dx$$

II. Вычислить определенный интеграл.

$$1. \int_2^7 \frac{\sqrt{x+2} dx}{x}.$$

$$8. \int_{\frac{1}{4}}^0 \frac{dx}{1+\sqrt{3x+1}}.$$

$$2. \int_{-\frac{3}{4}}^0 \frac{3x dx}{\sqrt{(x+1)^3}}.$$

$$9. \int_{-14}^0 \frac{dx}{\sqrt[3]{x^2}}.$$

$$3. \int_0^1 \frac{\sqrt{x} dx}{4-x}.$$

$$10. \int_0^4 \frac{\sqrt{x} dx}{4+x}.$$

$$4. \int_{-85}^0 \frac{dx}{\sqrt[3]{x^2}}.$$

$$11. \int_3^6 \frac{\sqrt{x-3} dx}{x}.$$

$$5. \int_0^4 \frac{dx}{\sqrt{x-3}}.$$

$$12. \int_0^3 \frac{(x^2+\sqrt{1+x}) dx}{\sqrt{1+x}}.$$

$$6. \int_{-4}^1 \frac{x dx}{\sqrt{(5-x)^3}}.$$

$$13. \int_{21}^{11} \frac{\sqrt{x-2} dx}{21+\sqrt{x-2}}.$$

$$7. \int_{\frac{3}{4}}^0 \frac{dx}{2-\sqrt{1+x}}.$$

$$14. \int_{25}^{49} \frac{\sqrt{x} dx}{x-6}.$$

15.  $\int_{-18}^1 \frac{dx}{\sqrt[3]{x^2}}$ .

16.  $\int_2^7 \frac{x dx}{(x^2 + 1)^2}$ .

17.  $\int_1^e \frac{(1 + \lg x) dx}{x}$ .

18.  $\int_1^{e^3} \frac{dx}{x\sqrt{1 + \ln x}}$ .

19.  $\int_{-5}^{-3} \frac{dx}{(11 + 5x)^3}$ .

20.  $\int_2^9 \frac{(y-1) dy}{\sqrt{y+1}}$ .

21.  $\int_0^{16} \frac{dx}{\sqrt{x+9} - \sqrt{x}}$ .

22.  $\int_0^{2a} \frac{dx}{2a-x} \quad (a > 0)$ .

23.  $\int_1^e \frac{dx}{x\sqrt{1 - (\ln x)^2}}$ .

24.  $\int_1^2 \frac{e^{1/x} dx}{x^2}$ .

25.  $\int_2^3 \frac{dx}{2x^2 + 3x - 2}$ .

26.  $\int_0^1 \frac{dx}{x^2 + 4x + 5}$ .

27.  $\int_0^1 (e^x - 1)^7 e^x dx$

28.  $\int_{-0.5}^1 \frac{dx}{\sqrt{8 + 2x - x^2}}$ .

29.  $\int_0^{\pi/2} \cos^5 x \sin 2x dx$

30.  $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x} dx$

31.  $\int_{1/\pi}^{2/\pi} \frac{\sin 1/x}{x^2} dx$

32.  $\int_0^1 x e^x dx$

33.  $\int_0^{\pi/2} x \cos x dx$

34.  $\int_0^{e-1} \ln(x+1) dx$

35.  $\int_0^3 2^{x-1} dx$ .

36.  $\int_0^1 \frac{x^2 dx}{1+x^6}$ .

37.  $\int_0^{\pi/4} \sin^2 \varphi d\varphi$

38.  $\int_0^1 \frac{dx}{4x^2 + 4x + 5}$ .

39.  $\int_1^2 \frac{dx}{x+x^2}$ .

40.  $\int_1^2 e^{1/x^2} \frac{dx}{x^3}$ .

41.  $\int_0^{\pi/2} \cos^3 \varphi d\varphi$

42.  $\int_{\pi/4}^{\pi/3} \frac{\sin^3 x}{\cos^7 x} dx$

43.  $\int_0^1 \frac{dx}{\sqrt{x^2 + 2x + 2}}$ .

44.  $\int_1^e \frac{\cos(\ln x) dx}{x}$ .

45.  $\int_{3/4}^2 \frac{dx}{\sqrt{2 + 3x - 2x^2}}$ .

||| Вычислить площадь плоской фигуры, ограниченной заданными кривыми. Сделать чертеж области.

1.  $3x^2 - 4y = 0, 2x - 4y + 1 = 0.$
2.  $3x^2 + 4y = 0, 2x - 4y - 1 = 0.$
3.  $2x + 3y^2 = 0, 2x + 2y + 1 = 0.$
4.  $3x^2 - 4y = 0, 2x + 4y - 1 = 0.$
5.  $3x^2 + 4y = 0, 2x + 4y + 1 = 0.$
6.  $2x - 3y^2 = 0, 2x + 2y - 1 = 0.$
7.  $3x^2 - 2y = 0, 2x - 2y + 1 = 0.$
8.  $4x + 3y^2 = 0, 4x + 2y + 1 = 0.$
9.  $3x^2 - 2y = 0, 2x + 2y - 1 = 0.$
10.  $4x - 3y^2 = 0, 4x + 2y - 1 = 0.$
11.  $y = x^3 + 3, x = 0, y = x - 1, x = 2.$
12.  $y = x^3 + 2, x = 0, y = x - 2, x = 2.$
13.  $y = x^3 + 1, x = 0, y = x - 3, x = 2.$
14.  $y = x^3 - 1, x = 0, y = x - 5, x = 2.$
15.  $y = x^3 - 2, x = 0, y = x - 6, x = 2.$
16.  $y = x^2, y = \sqrt{x}$
17.  $y^2 + 8x = 16, y^2 - 24x = 48$
18.  $y = x^2, y = \frac{x^3}{3}$
19.  $y^2 = 4x, x^2 = 4y$
20.  $y = x^2 + 2x, y = x + 2$
21.  $y = -x^2, x + y + 2 = 0$
22.  $2x^2 - y = 0, 3x + y - 2 = 0$
23.  $y = \ln(x - 1), x = 4, y = 0$

24.  $y^2 = 4x^3, y = 2x^2$
25.  $y = \sin x, y = \cos x, x = 0$
26.  $y = 0,25x^2, y = 3x - 0,5x^2$
27.  $x^2 - 6x + y = 0, x + y = 6$
28.  $y = 2x^2, 5x + y + 2 = 0$
29.  $y = 2x - x^2, x + y = 0$
30.  $y = 2^x, y = 2, x = 0$
31.  $y = 2x^2 + x + 1, y = 0, x = 3, x = 1$
32.  $y = \frac{1}{2}x^2 + 1, x + \frac{5}{2} = y$
33.  $y = x^2 - 2x + 2, y = 2 + 4x - x^2$
34.  $y = \ln(x + 6), y = 3 \ln x, x = 0, y = 0$
35.  $y = \operatorname{tg} x, y = \frac{2}{3} \cos x, x = 0$
36.  $y = x^3, y = x^{1/3}, x = 0, x = 1$
37.  $y = 2x^2 + 1, y = 0, x = -1, x = 1$
38.  $y = 2\sqrt{x}, 6 - y = 0, x = 0$
39.  $y = 3^x, y = 0, x = 3, x = -1$
40.  $y = \sin \frac{x}{3}, x = -\pi, x = 2\pi, y = 0$
41.  $y = x^4 - 2x^2 + 5, y = 1, x = 0, x = 1$
42.  $y = 4x - x^2, y = 0, x = 2$
43.  $y = \frac{9}{x}, y = x, x = 9, y = 0$
44.  $y = x^4 - 10x^2 + 9, x = 0, y = 0$
45.  $y = x^2 - 2x + 3, y = 3x - 1$