ДОМАШНЕЕ ЗАДАНИЕ № 5.

«Функции многих переменных. Кратные и криволинейные интегралы.»

1. Дана функция $z = f(x, y)$. Показать, что

$$F\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y^2}, \frac{\partial^2 z}{\partial x \partial y}\right) = 0.$$

1.1. $z = \frac{x}{y}$; $F = x\frac{\partial^2 z}{\partial x \partial y} - \frac{\partial z}{\partial y}$.

1.2. $z = x^y$; $F = y\frac{\partial^2 z}{\partial x \partial y} - (1 + y\ln x)\frac{\partial z}{\partial x}$.

1.3. $z = xe^{\frac{y}{x}}$; $F = x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2}$.

1.4. $z = \sin(x + ay)$; $F = \frac{\partial^2 z}{\partial y^2} - a^2\frac{\partial^2 z}{\partial x^2}$.

1.5. $z = \cos y + (y - x)\sin y$; $F = (x - y)\frac{\partial^2 z}{\partial x \partial y} - \frac{\partial z}{\partial y}$.

1.6. $z = \frac{y}{(x^2 - y^2)^5}$; $F = \frac{1}{x}\frac{\partial z}{\partial x} + \frac{1}{y}\frac{\partial z}{\partial y} - \frac{z}{y^2}$.

1.7. $z = \frac{y^2}{3x} + \arcsin(xy)$; $F = x^2\frac{\partial z}{\partial x} - xy\frac{\partial z}{\partial y} + y^2$.

1.8. $z = \ln(x^2 + y^2 + 2x + 1)$; $F = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$.

1.9. $z = e^{xy}$; $F = x^2\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} + 2xyz$.

1.10. $z = \ln(x + e^{-y})$; $F = \frac{\partial z}{\partial x}\frac{\partial^2 z}{\partial x \partial y} - \frac{\partial z}{\partial y}\frac{\partial^2 z}{\partial x^2}$.

1.11. $z = e^x(x\cos y - y\sin y)$; $F = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$.

1.12. $u = \frac{e^{-2} + e^2}{r}$, где $r = \sqrt{x^2 + y^2 + z^2}$,

$$F = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}.$$

1.13. $z = xchx\sin y + yshx\cos y$; $F = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$.

1.14. $z = arctg\frac{y}{x}$; $F = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$.

1.15. $u = \frac{1}{r}$, где $r = \sqrt{x^2 + y^2 + z^2}$,

$$F = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}.$$

1.16. $z = \ln r$, где $r = \sqrt{x^2 + y^2}$, $F = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$.

1.17. $z = \frac{1}{2\sqrt{\pi x}}e^{-y^2/4x}$, $F = \frac{\partial z}{\partial x} - \frac{\partial^2 z}{\partial y^2}$.

1.18. $z = \frac{1}{y^2 - x^2}$, $F = \frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} + 4z^2$.

1.19. $z = \sin y\cos x$; $F = \frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2}$.

1.20. $u = \frac{1}{\sqrt{x^2 - y^2 - z^2}}$, $F = \frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} - \frac{\partial^2 u}{\partial z^2}$.

1.21. $z = x\sin(x+y) + y\cos(x+y)$;

$$F = \frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2}.$$

1.22. $z = \sin(2x+y)e^{\frac{y}{2}-x} + \cos(y-2x)$;

$$F = \frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial y^2} + 2\frac{\partial z}{\partial x} + 4\frac{\partial z}{\partial y}.$$

1.23. $z = \sin xy + \sqrt{xy}\cos{}^{y}\!/_{x}$;

$$F = x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2}.$$

1.24. $z = xe^{x+y} + y\sin(x+y)$;

$$F = \frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2}.$$

1.25. $z = (2x+y)^2 e^{\frac{y}{2}-x} + (y-2x)^2$;

$$F = \frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial y^2} + 2\frac{\partial z}{\partial x} + 4\frac{\partial z}{\partial y}.$$

1.26. $z = (xy)^2 + \sqrt{xy}e^{x/y}$; $F = x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2}$.

1.27. $z = \sin xy + \cos{}^{y}\!/_{x}$;

$$F = x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} - y\frac{\partial z}{\partial y} + x\frac{\partial z}{\partial x}.$$

1.28. $z = e^x\cos y$; $F = z\frac{\partial^2 z}{\partial x \partial y} - \frac{\partial z \partial z}{\partial x \partial y}$.

1.29. $z = x\cos(x+y) + y(x+y)^3$;

$$F = \frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2}.$$

1.30. $z = (y+2x)^3 e^{\frac{y}{2}-x} + \sin(y-2x)$;

$$F = \frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial y^2} + 2\frac{\partial z}{\partial x} + 4\frac{\partial z}{\partial y}.$$

1.31. $z = e^{xy} + \sqrt{xy}\sin y/x$;

$$F = x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2}.$$

1.32. $z = e^{xy} - \cos y/x$;

$$F = x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} - y\frac{\partial z}{\partial y} + x\frac{\partial z}{\partial x}.$$

1.33. $z = \sin xe^{-y}$;

$$F = z\frac{\partial^2 z}{\partial x \partial y} - \left(\frac{\partial z}{\partial x}\right)\cdot\left(\frac{\partial z}{\partial y}\right).$$

1.34. $z = \sin\frac{y}{x} + xe^{y/x}$;

$$F = x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2}.$$

1.35. $z = \frac{1}{y}\left[\sin(x+y) + \cos(x-y)\right]$.

$$F = \frac{\partial^2 z}{\partial x^2} - \frac{1}{y^2}\frac{\partial}{\partial y}\left(y^2\frac{\partial z}{\partial y}\right).$$

1.36. $z = \frac{1}{x}\left[\sin(x-y) + \cos(x+y)\right]$.

$$F = \frac{\partial}{\partial x}\left(x^2\frac{\partial z}{\partial x}\right) - x^2\frac{\partial^2 z}{\partial y^2}.$$

1.37. $z = \sqrt{x+y} + y\cos(x+y)$;

$$F = \frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2}.$$

1.38 $z = \ln(e^{-x} + y)$;

$$F = \frac{\partial z}{\partial y} \cdot \frac{\partial^2 z}{\partial x \partial y} - \frac{\partial z}{\partial x} \cdot \frac{\partial^2 z}{\partial y^2}.$$

1.39 $z = \frac{1}{y}\left[\cos(x+y) + e^{x-y}\right]$,

$$F = \frac{\partial^2 z}{\partial x^2} - \frac{1}{y^2}\frac{\partial}{\partial y}\left(y^2 \frac{\partial z}{\partial y}\right).$$

1.40 $z = \frac{1}{x}\left[2\sin(x-y) + e^{x+y}\right]$,

$$F = \frac{\partial}{\partial x}\left(x^2 \frac{\partial z}{\partial x}\right) - x^2 \frac{\partial^2 z}{\partial y^2}.$$

2. Даны функция $z = z(x, y)$, точка $A(x_0, y_0)$ и вектор $\bar{a}$.

Найти:

1) $\overline{grad\, z}$ в точке A;

2) производную в точке A в направлении вектора $\bar{a}$.

2.1. $z = 2x^2 + xy$; $A(-1,2)$; $\bar{a} = 3\bar{i} + 4\bar{j}$.

2.2. $z = arctg\frac{y}{x}$; $A(-1,-1)$; $\bar{a} = \bar{i} - \bar{j}$.

2.3. $z = x^3 y + xy^2$; $A(1,3)$; $\bar{a} = -5\bar{i} + 12\bar{j}$.

2.4. $z = \ln(2x + 3y)$; $A(2,2)$; $\bar{a} = 2\bar{i} - 3\bar{j}$.

2.5. $z = 5x^2 y + 3xy^2$; $A(1,1)$; $\bar{a} = 6\bar{i} - 8\bar{j}$.

2.6. $z = \frac{3x}{y^2}$; $A(3,4)$; $\bar{a} = -3\bar{i} - 4\bar{j}$.

2.7. $z = arctg(xy)$; $A(2,3)$; $\bar{a} = 4\bar{i} + 3\bar{j}$.

2.8. $z = \ln(3x^2 + 2xy^2)$; $A(1,2)$; $\bar{a} = 3\bar{i} - 4\bar{j}$.

2.9. $z = \frac{x+y}{x^2+y^2}$; $A(1,-2)$; $\bar{a} = \bar{i} + 2\bar{j}$.

2.10. $z = 5x^2 - 2xy + y^2$; $A(1,1)$; $\bar{a} = 2\bar{i} - \bar{j}$.

2.11. $z = 1 + x^2 y^3$; $A(1,1)$; $\bar{a} = \bar{i} - 3\bar{j}$.

2.12. $z = 3x^2 + 5y^2$; $A(-1,2)$; $\bar{a} = -\frac{1}{\sqrt{2}}\bar{i} + \frac{1}{\sqrt{2}}\bar{j}$.

2.13. $z = x\sin(x+y)$; $A\left(\frac{\pi}{4};\frac{\pi}{4}\right)$; $\bar{a} = 3\bar{i} + \bar{j}$.

2.14. $z = x^3 + 2xy^2$; $A(2/3,1/3)$; $\bar{a} = 3\bar{i} + \bar{j}$.

2.15. $z = \ln(x^2 + y^2)$; $A(1,2)$; $\bar{a} = -\frac{1}{3}\bar{i} + \frac{2}{3}\bar{j}$.

2.16. $z = \arcsin\frac{x}{y}$; $A(1/4,3/4)$; $\bar{a} = \frac{1}{\sqrt{2}}\bar{i} + \frac{1}{\sqrt{2}}\bar{j}$.

2.17. $z = arctg(xy)$; $A(0,1)$; $\bar{a} = \frac{1}{3}\bar{i} + \bar{j}$.

2.18. $z = \ln(1 - x^2 + y^2)$; $A(1,2)$; $\bar{a} = 2\bar{i} - \bar{j}$.

2.19. $z = e^{xy-y^2}$, $A(-1;3)$, $\bar{a} = 5\bar{i} + 2\bar{j}$

2.20. $z = 5x^2 + \cos(x-y)$; $A(2,3)$; $\bar{a} = 6\bar{i} + 2\bar{j}$.

2.21. $z = arctg\frac{y^2}{x}$; $A(-2,2)$; $\bar{a} = 6\bar{i} - \bar{j}$.

2.22. $z = tg(xy)$; $A(2,1)$; $\bar{a} = \frac{1}{2}\bar{i} + \frac{3}{8}\bar{j}$.

2.23. $z = \ln(e^x + e^y)$; $A(-1,4)$; $\bar{a} = 3\bar{i} + 4\bar{j}$.

2.24. $z = \frac{x+y}{x^2-y}$; $A(2,1)$; $\bar{a} = \frac{1}{\sqrt{2}}\bar{i} - \frac{1}{\sqrt{2}}\bar{j}$.

2.25. $z = xe^{\sin xy}$; $A(1,-\pi)$; $\bar{a} = \frac{1}{\sqrt{2}}\bar{i} + \frac{1}{\sqrt{2}}\bar{j}$.

2.26. $z = \sin x - x^2 y$; $A(2\pi,1)$; $\bar{a} = 2\bar{i} + \bar{j}$.

2.27. $z = \frac{x(x-y)}{y^2}$; $A(3,-4)$; $\bar{a} = -3\bar{i} + 5\bar{j}$.